Application of scaling equations to deal with the spatial aggregation effect on watershed hydrological modelling

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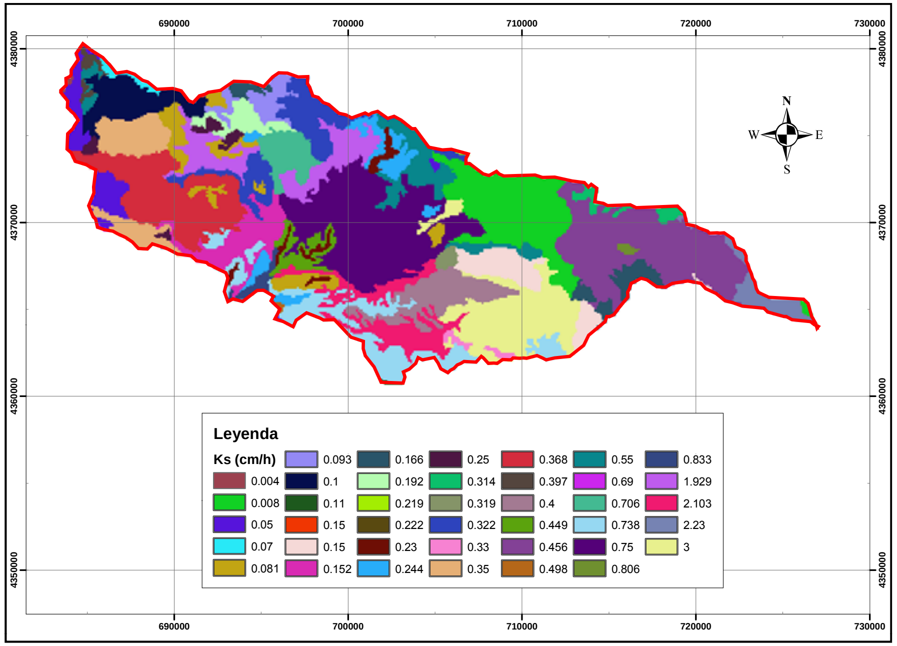
Spatial scales in soil characteristics

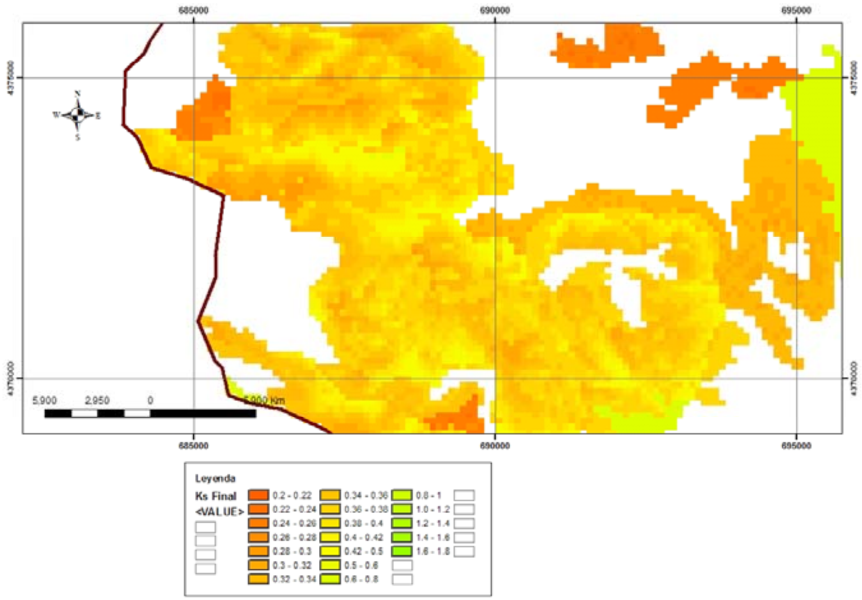
- **Organization**, in the minor scales (Blöschl and Sivapalan, 1995)
  - General pattern can be explained deterministically by a few number of factors
  - It is possible to delineate Cartographic Units, but
    - Subjective
    - Scale dependent
  - With modal values



E.g.: Modal values of vertical saturated permeability in Rambla del Poyo, Spain

Spatial scales in soil characteristics

- **Randomness**, in the larger scales
  - Detailed structure is due to a large number of factors
  - Stochastic models with spatial dependence structure

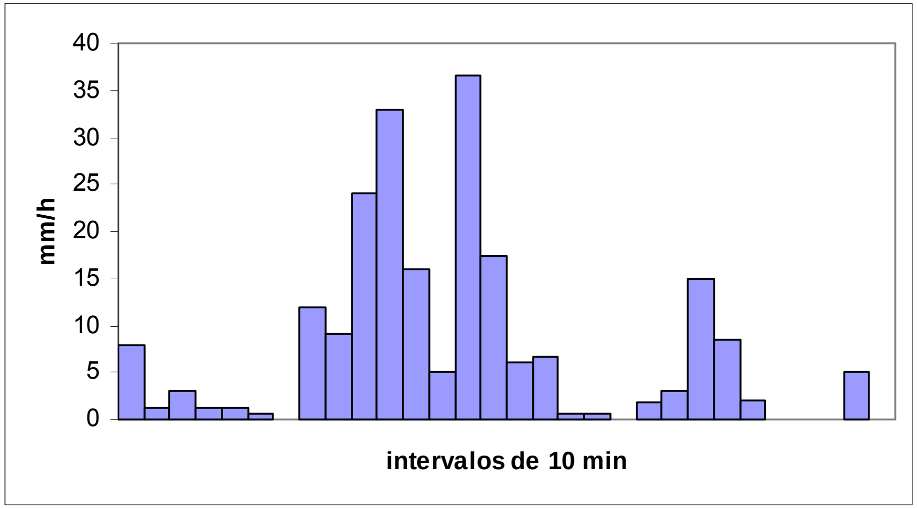


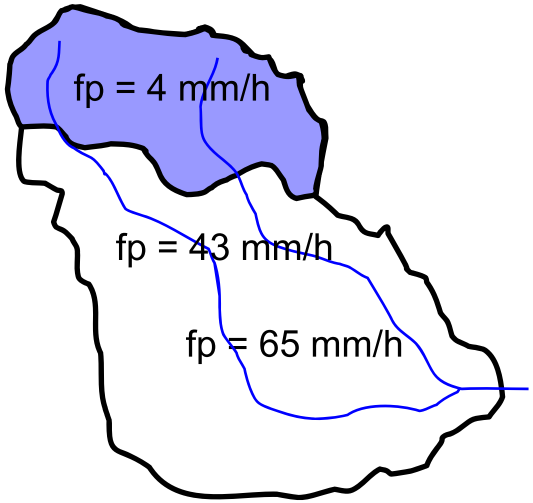
E.g.: Vertical saturated permeability at each cell in one Cartographic Unit of Rambla del Poyo, Spain

Problems with heterogeneity

- Scale effects when averaging non linear processes:

- Spatial aggregation





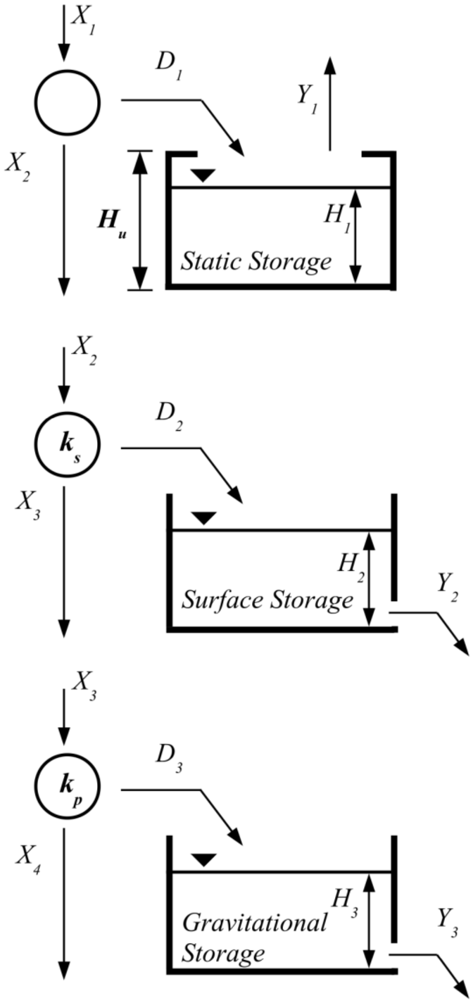
Effective parameters

- Scale effects when averaging non linear processes:

  The mean process is not the result with the mean parameter and/or input

- **Effective parameter**: parameter value which reproduces the mean process at the mesoscale, but:
  - Different to the spatial mean of the point scale values
    - They can lose their physical meaning at the meso or macroscale
  - **Non-stationary**

Non-linear processes



- Static Storage

Water excedence: $X_2 = Max[0; X_1 - H_u + H_1]$

Capillary infiltration: $D_1 = X_1 - X_2$

Evapotranspiration: $Y_1 = Min[ETP \cdot \lambda; H_1]$

- Surface Storage

Gravitational infiltration: $X_3 = Min[X_2; \Delta t \cdot k_s]$

Gravitational Storage

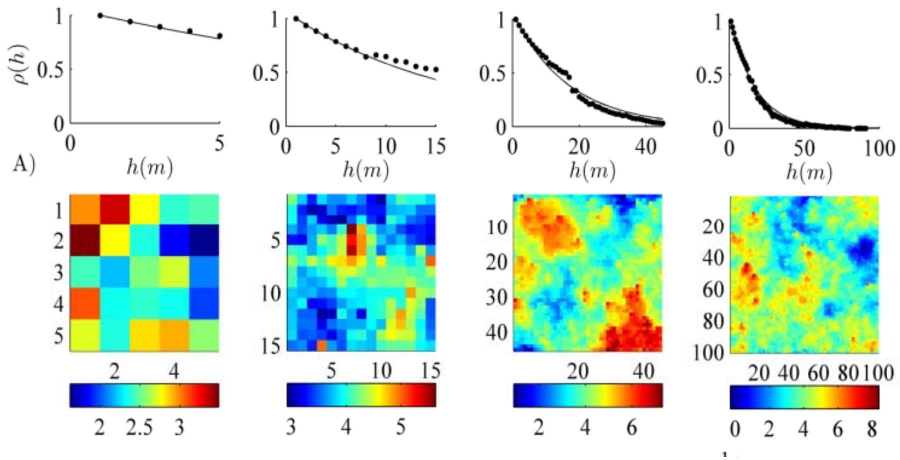
Percolation: $X_4 = Min[X_3; \Delta t \cdot k_p]$

Synthetic heterogeneity

- Generation of random parameter fields (***H_u***, ***k_s*** and ***k_p***)
  - PDF of H_u [Beta(a,b)]: $$f = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} H_u^{a-1}(1-H_u)^{b-1}$$
  - PDF of k_s and k_p [LN(μ,σ)]: $$f = \frac{1}{k_s \sigma \sqrt{2\pi}} e^{\left[-\frac{(\ln k_s - \mu)^2}{2\sigma^2}\right]}$$
  - Exponential spatial autocorrelation: $$\rho(h) = e^{\left(-\frac{3h}{a}\right)}$$

- Sampling algorithm (Pinder and Celia, 2006):
  - Latin Hypercube Sampling
  - Cholesky Factorization

Synthetic heterogeneity



Statistics of the random fields

μ (H_u)	μ (k_s)	μ (k_p)	CV=σ/μ
70 100	20 60	2 6	0.5
			1
			1.5
			2

18 Correlation lengths: ***a*** = 2.5, 5, 10,… 50, 75, 100, 150, 250, 500, 2500 and 5000 m

Spatial scales

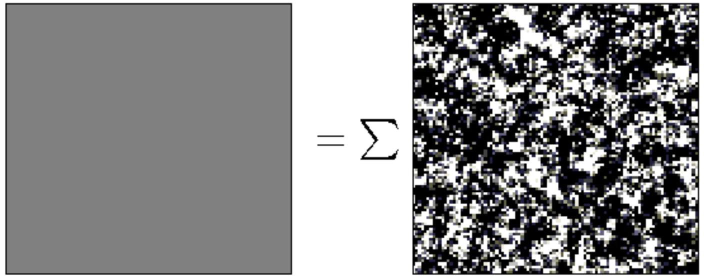
Microscale S1	Mesoscale S2		# of realizations
	Size	Notation	
[m²]	[m²]		
1 x 1	5 x 5	S2a	500
1 x 1	15 x 15	S2b	500
1 x 1	45 x 45	S2c	2500
1 x 1	100 x 100	S2d	5000

Scale effect: aggregation

- Aggregation:

Excedence $$X_2[S2] = \sum_{i=1}^{n} X_{2i}$$

Gravitational infiltration $$X_3[S2] = \sum_{i=1}^{n} X_{3i}$$



Flow aggregation

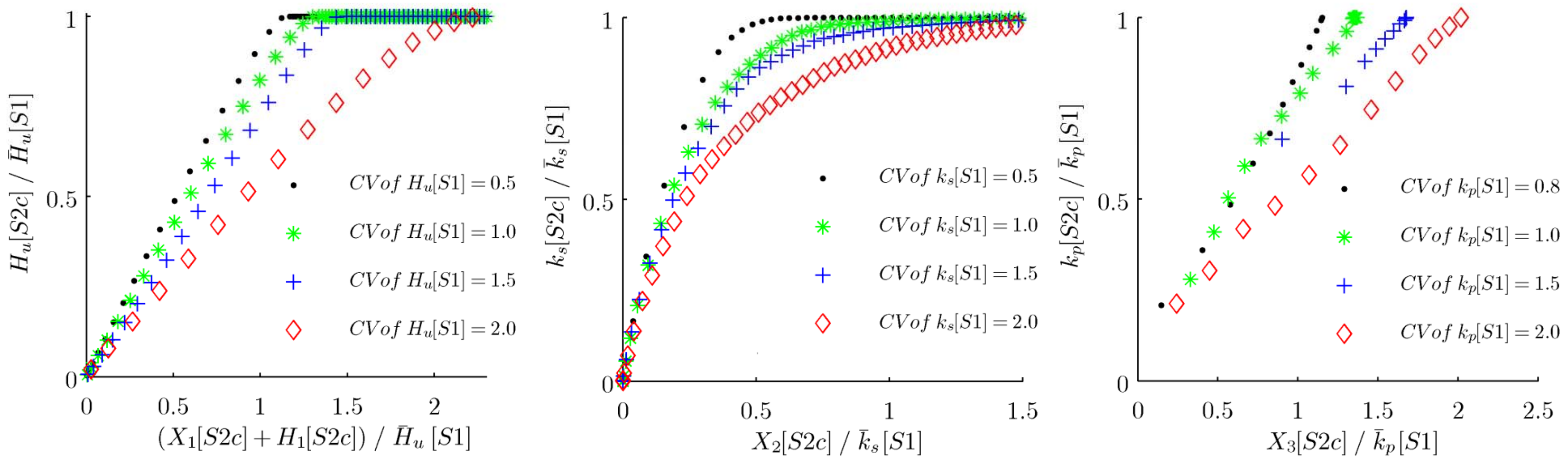
- Mesoscale effective parameters (inverse solution):

$$H_u[S2]_t = X_1[S2] + H_1[S2] - X_2[S2]$$

$$k_s[S2]_t = \begin{cases} X_2[S2]\cdot(\Delta t)^{-1} & X_3[S2] = X_2[S2] \\ X_3[S2]\cdot(\Delta t)^{-1} & X_3[S2] > X_2[S2] \end{cases}$$ (similar expression for k_p)

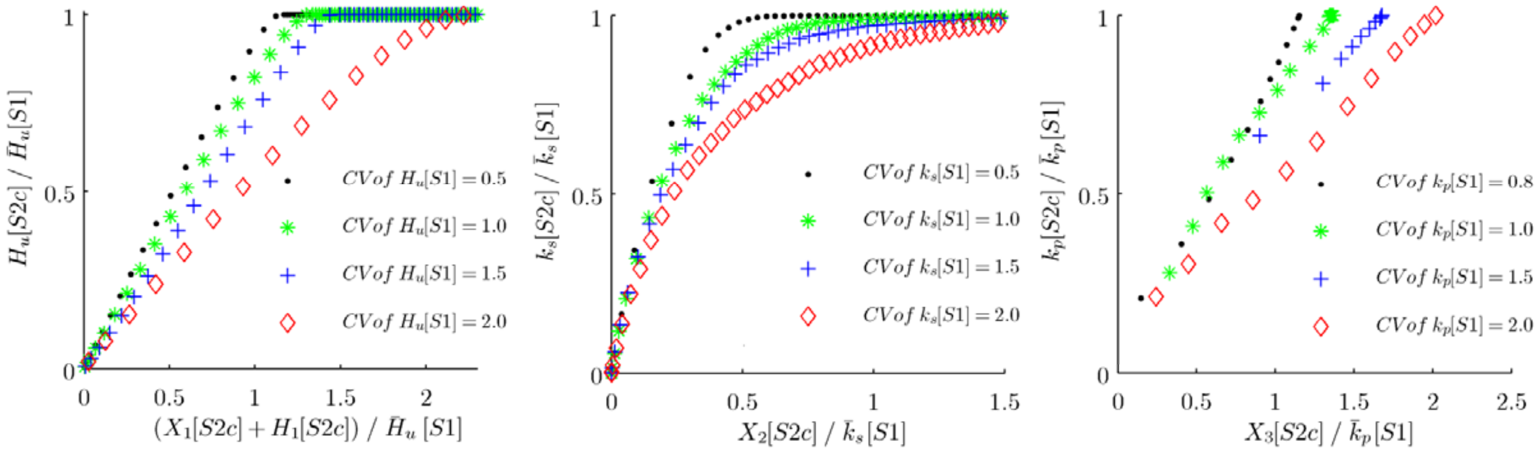
Mesoscale effective parameters

- H_u , k_s and k_p depend strongly on state variables and input and are sensible to microscale heterogeneity:



- Some sensitivity to CV
- Low sensitivity to the spatial dependence structure

Scaling equations



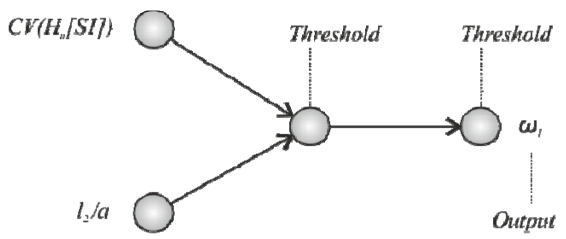
$$H_u[S2]_t = (X_{1t} + H_{1t-1})\left\{1 - \Phi\left[\frac{\ln(X_{1t} + H_{1t-1} - \omega_1)}{\omega_2}\right]\right\} + \bar{H}_u[S1]\left\{\Phi\left[\frac{\ln(X_{1t} + H_{1t-1} - \omega_1)}{\omega_2} - 0.93\omega_1^{-0.47}\omega_2\right]\right\}$$

$$k_s[S2]_t = \bar{k}_s[S1]\left\{1 - \varepsilon\left(X_{2t}[S2], \alpha\right)\right\} - X_{2t}[S2]\left\{\varepsilon\left(X_{2t}[S2], \alpha\right)\right\}$$

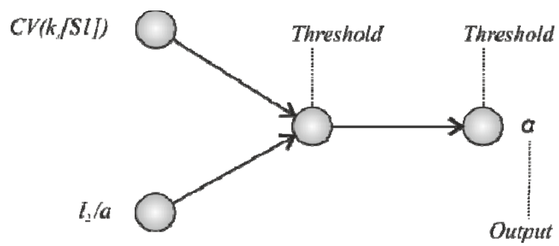
$$k_p[S2]_t = \bar{k}_p[S1]\left\{1 - \varepsilon\left(X_{3t}[S2], \beta\right)\right\} - X_{3t}[S2]\left\{\varepsilon\left(X_{3t}[S2], \beta\right)\right\}$$

Scaling equations

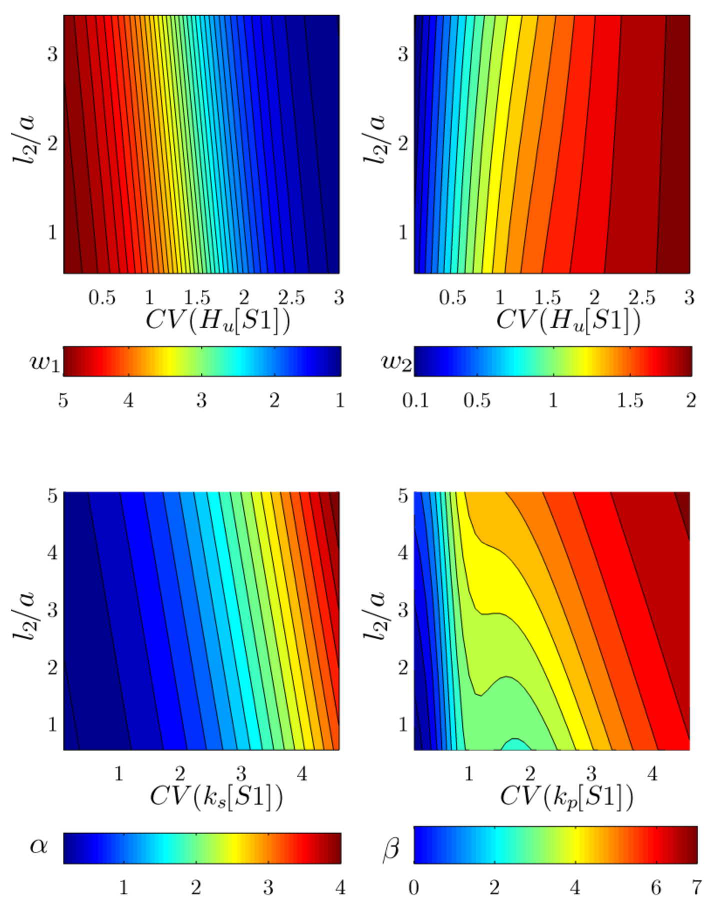
- The relationship between the microscale heterogeneity and the parameters of the scaling equations was estimated through multilayer perceptron neural networks:



RNA1	Hidden layer	Output
Transfer function	Hyperbolic tangent	Linear
Weights	-1.3787	0.9077
	-0.2293	
Threshold	-0.4837	0.2366

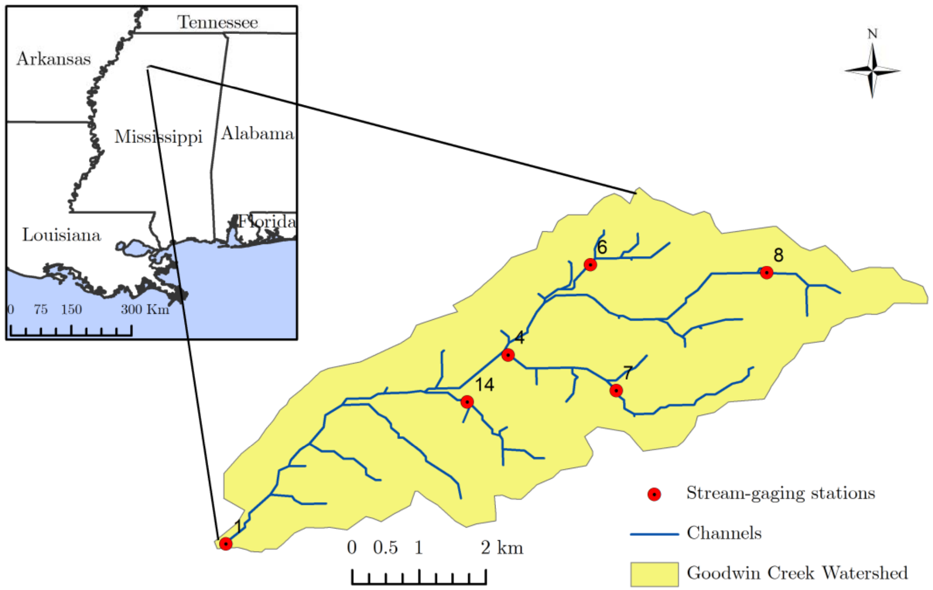


RNA3	Hidden layer	Output
Transfer function	Hyperbolic tangent	Linear
Weights	-0.6915	-1.2067
	-0.2664	
Threshold	0.6271	0.1874



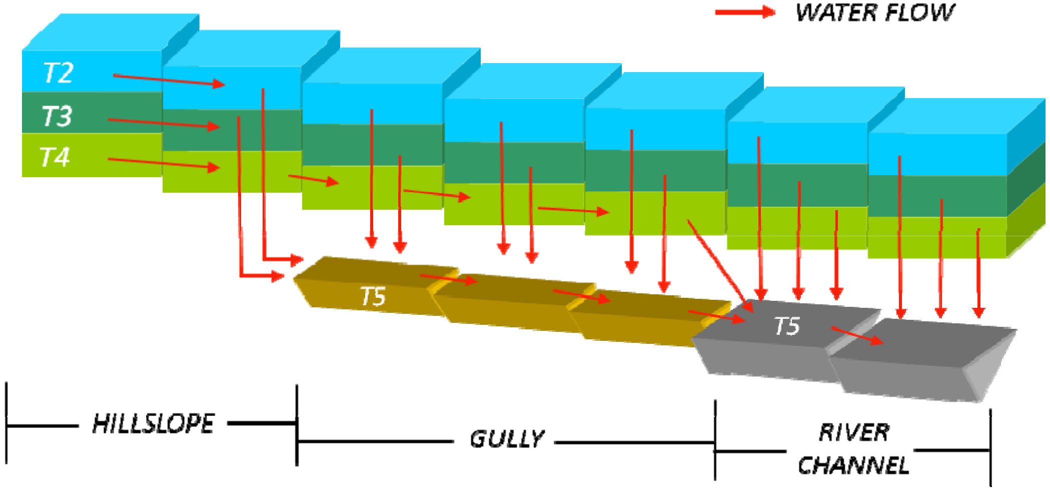
Study site

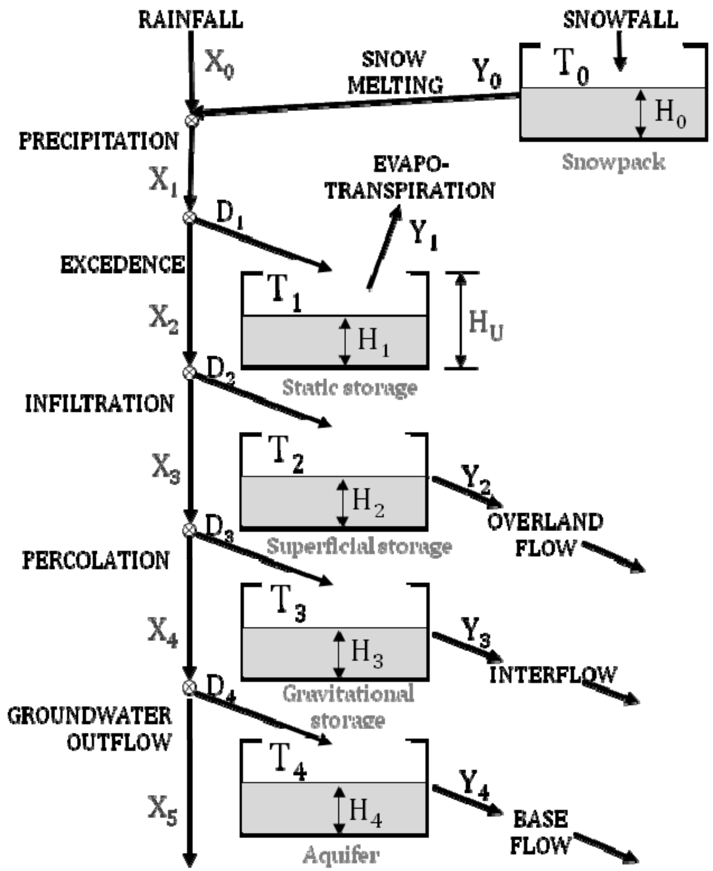
- **Goodwin Creek** experimental basin
  - Basin Area: 21.6 km^2
  - Ephemeral river
  - Hortonian runoff?
  - 16 raingauge stations (temporal resolution of 5 minutes)
  - 6 flowgauge stations (calibration at the outlet station)
  - DEM: 30x30 m^2
  - Five flood events were selected in the period of 1981 to 1983: peak flows from 38 to 106 m3/s



Basic hydrological model

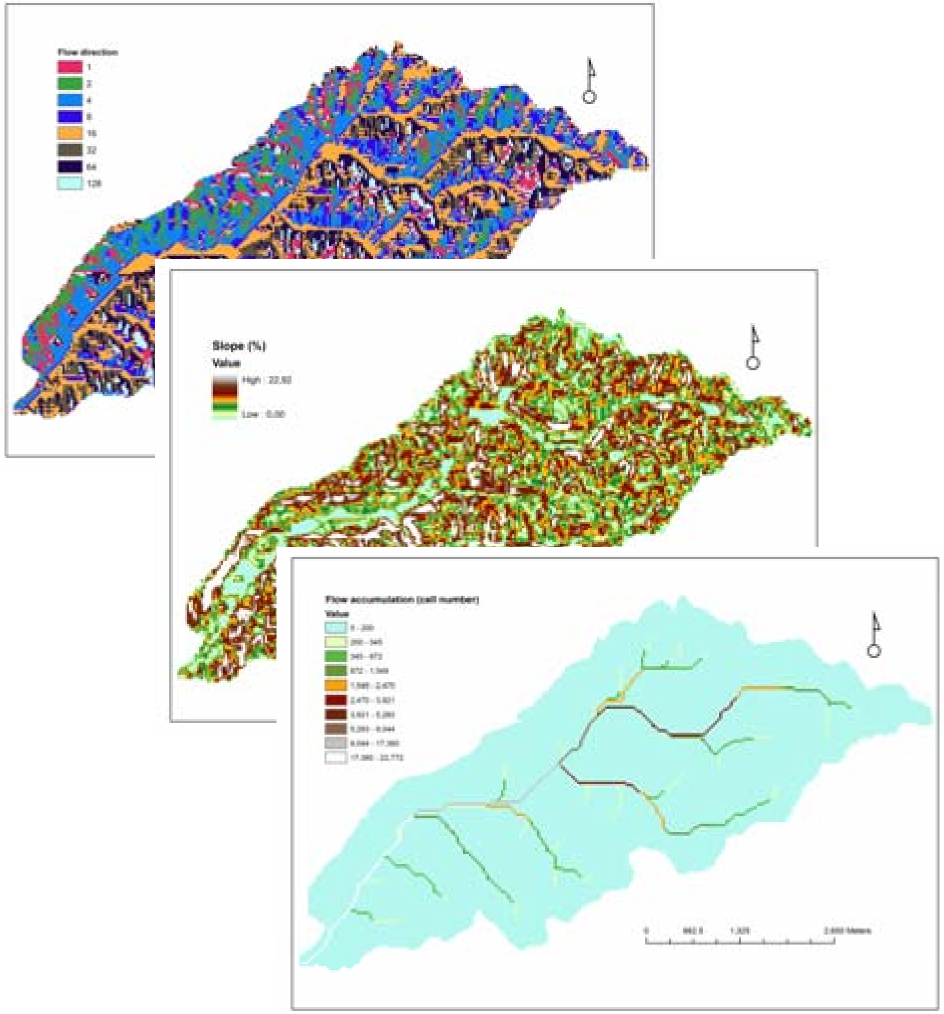
- **TETIS model**





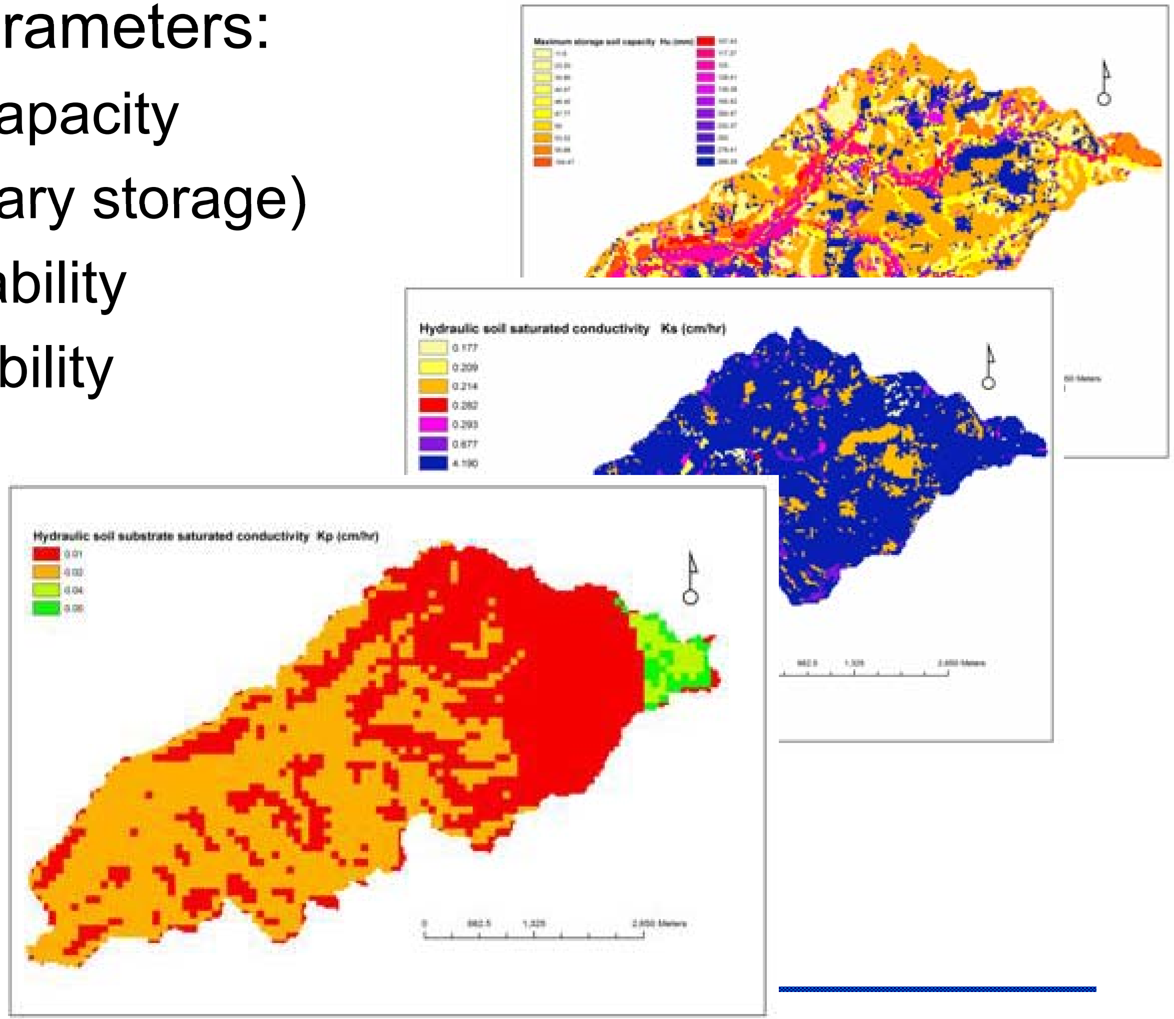
TETIS spatial information

- Derived from the DEM:
  - Flow direction
  - Slope
  - Flow accumulation



TETIS spatial information

- Hydrological soil parameters:
  - Static maximum capacity (interception + capillary storage)
  - Upper soil permeability
  - Substrate permeability

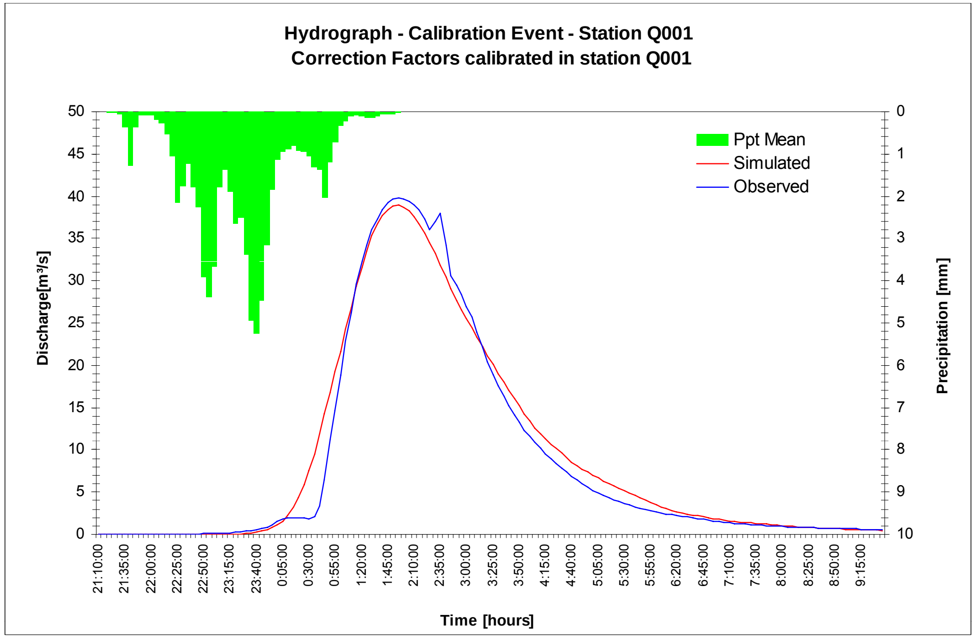


Hydrological automatic calibration

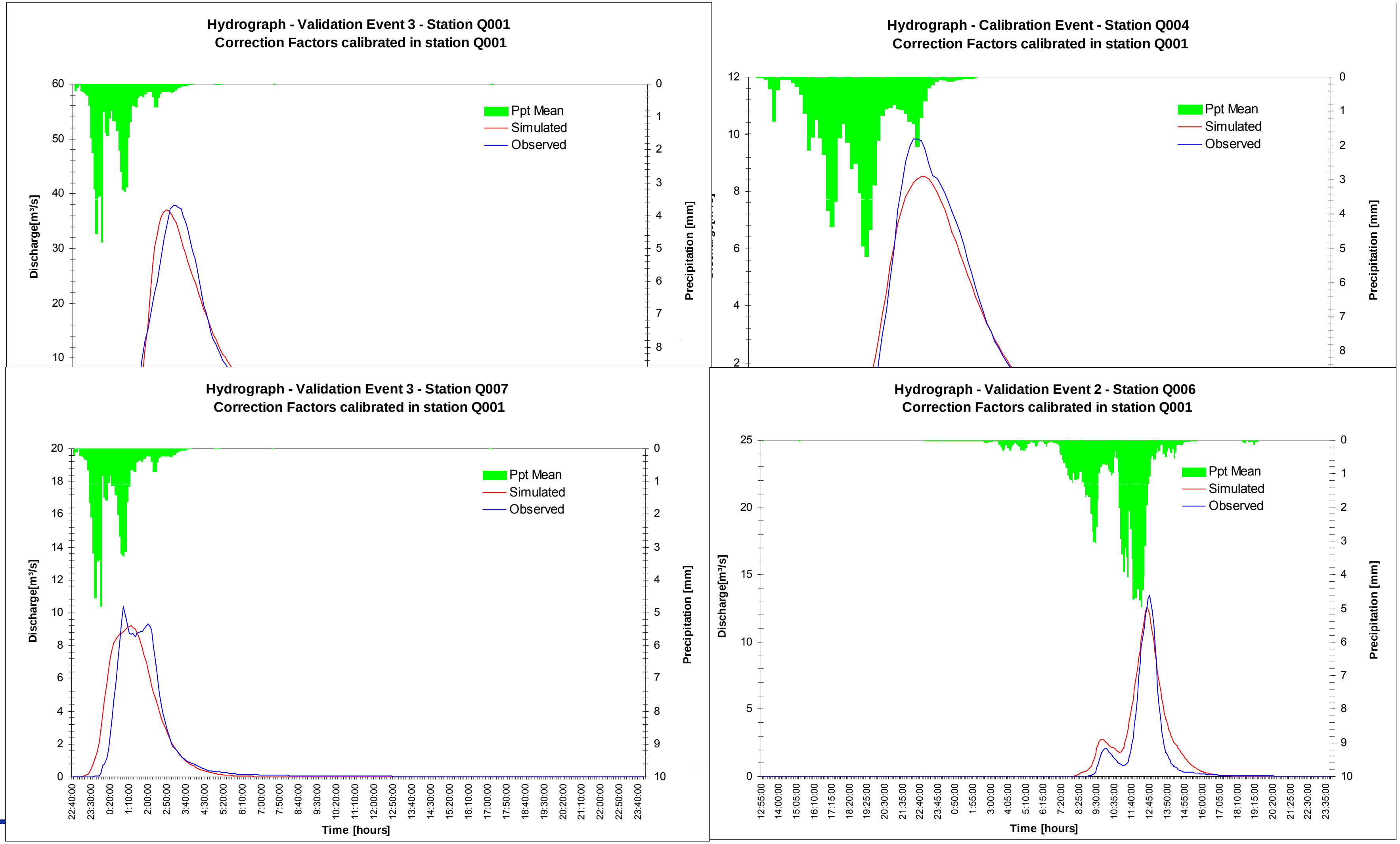
- Calibration of parameter maps correction factors & initial conditions
- Objective function: RMSE of the outlet disharges

- Streamflow:

basically overland flow!!



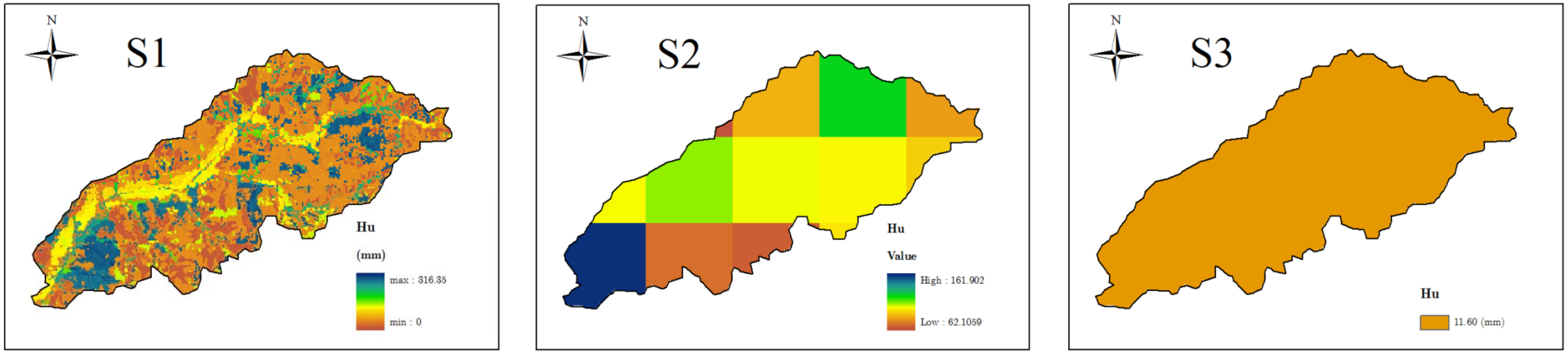
Hydrological validation



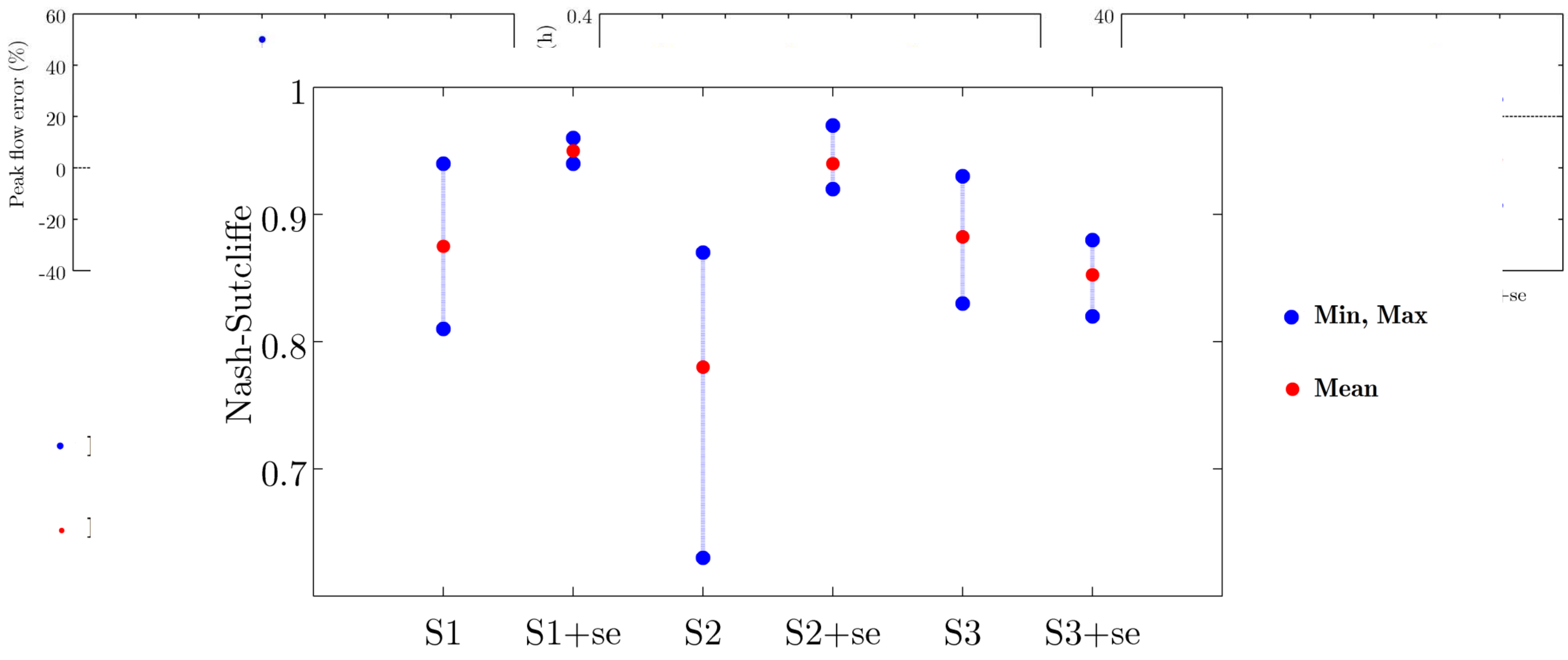
Modelling scenarios

- 3 information scales and with or w/o scaling equations

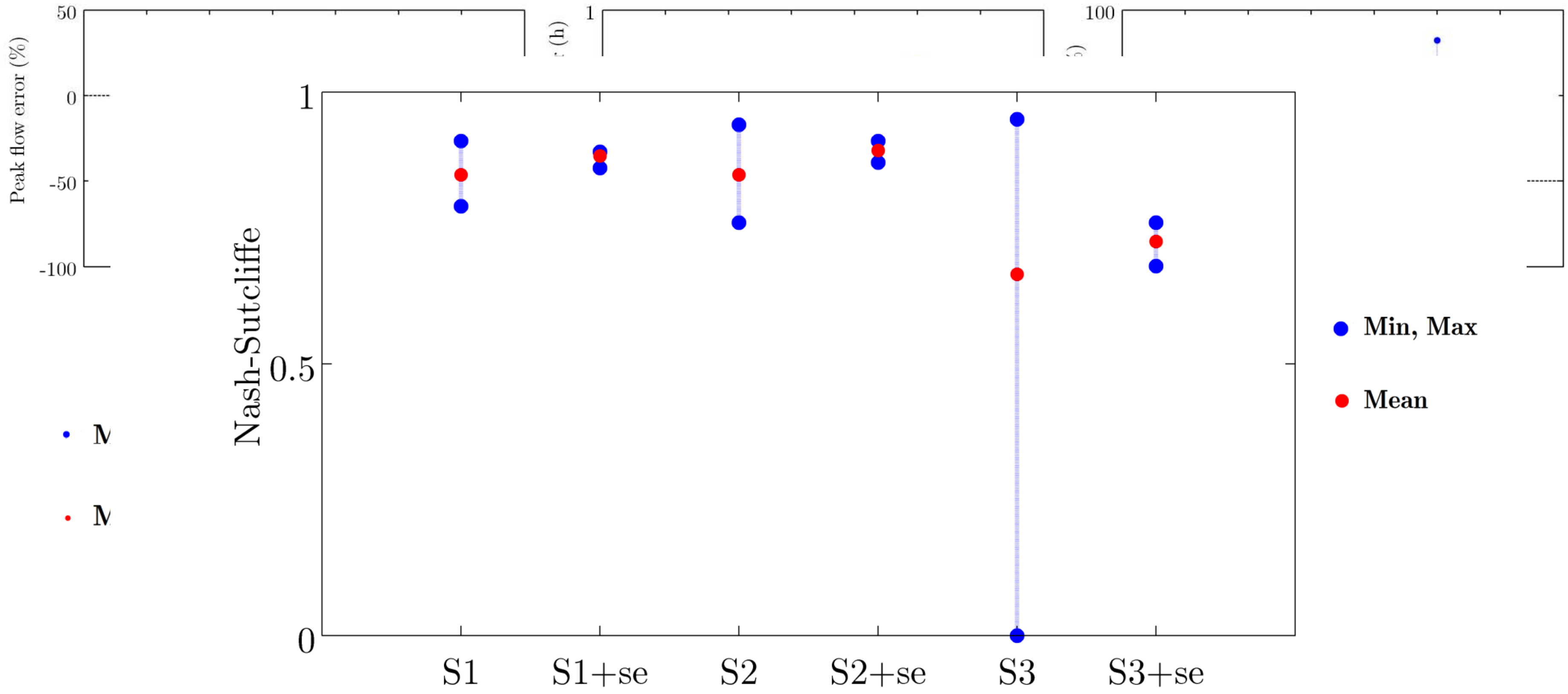
Notation	Scenarios
S1	Maps with resolution of 30x30 m^2, w/o scaling equations
S1+se	Maps with resolution of 30x30 m^2, with scaling equations
S2	Maps with resolution of 1740x1740 m^2, w/o scaling equations
S2+se	Maps with resolution of 1740x1740 m^2, with scaling equations
S3	Maps with the average for the whole catchment, w/o scaling equations
S3+se	Maps with the average for the whole catchment, with scaling equations



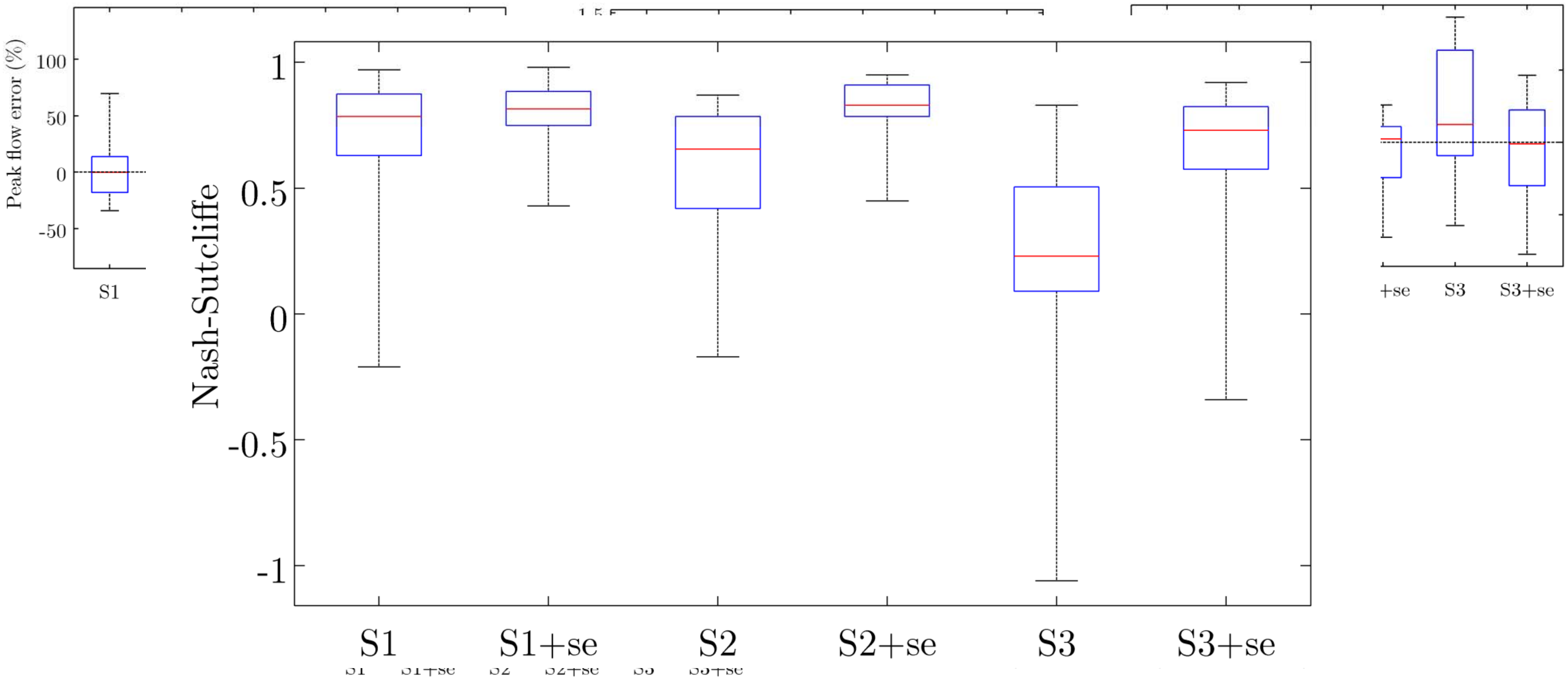
Results: spatial validation



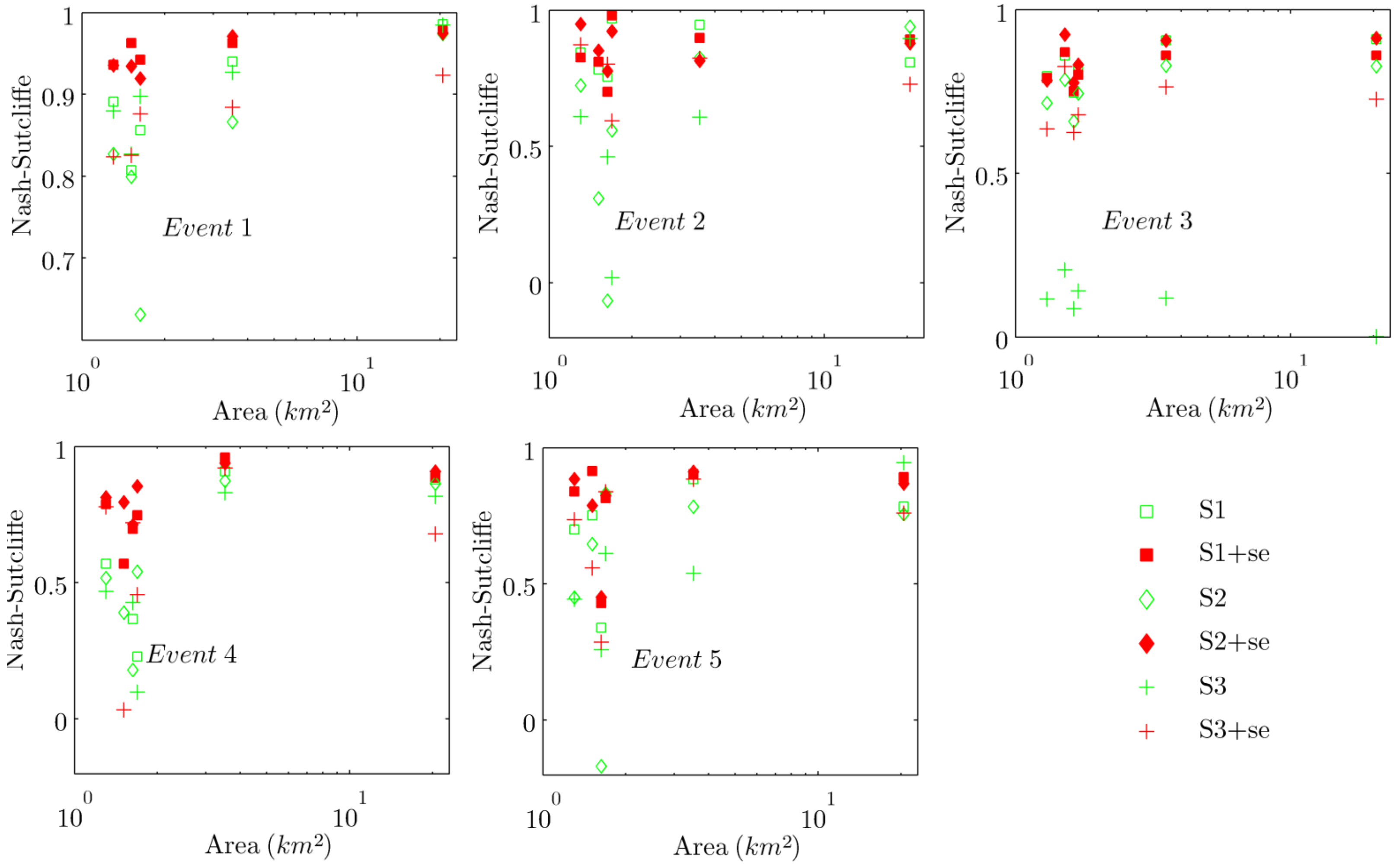
Results: temporal validation



Results: spatial-temporal validation



Results: performance vs. sub-basin area



Conclusions

- Non-linearities + parameter heterogeneity and/or input variability => non-stationary effective parameters

- It is important the sub-grid variability representation in hydrological modeling.

- Particularly, the use of scaling equations implies:
  - For all information scenarios, a significant model performance improvement in validations at internal flowgauges and for the smallest storm events.
  - A better performance of S1+se and S2+se in comparison to the reference model S1.

Acknowledgements

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